

*hSNS – Portuguese public hospital performance assessment using a multicriteria decision analysis framework* 

# A network Data Envelopment Analysis to estimate nations' efficiency on the fight against SARS-CoV-2

Duarte Caldeira Dinis\*, a

\*presenting author

Miguel Alves Pereira<sup>a, b</sup>

Diogo Cunha Ferreira<sup>b</sup>

José Rui Figueira<sup>a, b</sup>

<sup>a</sup>CEG-IST, Instituto Superior Técnico, Universidade de Lisboa <sup>b</sup>CERIS, Instituto Superior Técnico, Universidade de Lisboa

Organization









## Motivation

- The ongoing SARS-CoV-2 pandemic is deeply impacting health systems worldwide;
- COVID-19 is characterized by relatively high admission rates for patients, particularly those requiring intensive care unit (ICU) treatment (Remuzzi & Remuzzi, 2020), and long hospital stays (Rees et al., 2020);
- This drastically diminishes the response capacity of health services to provide consistent and appropriate care for COVID and non-COVID patients;
- It is, thus, important to assess the efficiency of different countries in responding to the current crisis, identifying which health systems are presenting the best/worst performances.

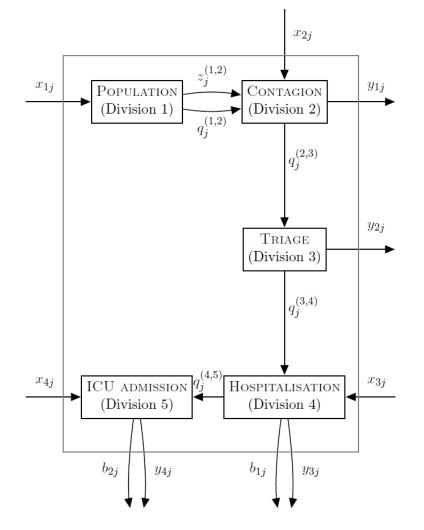


## Network Data Envelopment Analysis (DEA)

- Network DEA extends conventional DEA by considering the internal structures of the entities under analysis, the *decision-making units* (DMUs), for efficiency measurement;
- The approach allows the internal components of a DMU to be assessed by considering intermediate inputs (or products), and intermediate outputs, both desirable and undesirable;
- As with DEA, network DEA models can be oriented to output-maximization, or inputminimization.

## hSNS Workshop 26th February 2021 – Online

#### General series structure with five single-division stages



#### Inputs

$$\begin{split} x_{1j} &- health \ expenditure \ (M \in) \\ x_{2j} &- testing \ equipment \ (\epsilon) \\ x_{3j} &- disinfection \ and \ sterilisation \ (\epsilon) \\ x_{4j} &- oxygen \ therapy \ (\epsilon) \end{split}$$

## **Desirable intermediate products** $z_i^{(1,2)}$ – *population that use PPE*

#### Undesirable intermediate products

 $q_j^{(1,2)} - pop. that does not use PPE$  $q_j^{(2,3)} - infected pop.$  $q_j^{(3,4)} - infected pop. in hospital$  $q_j^{(4,5)} - hospital pop. in ICU$ 

#### Desirable outputs

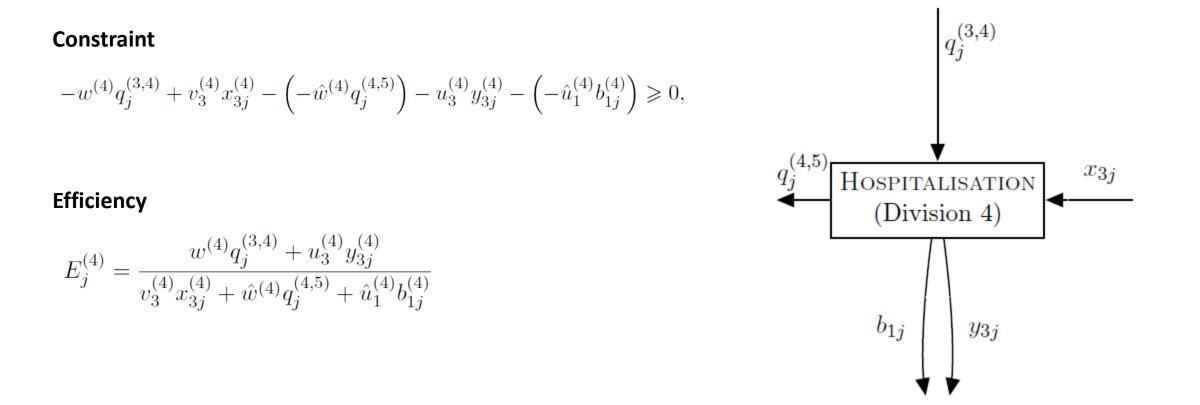
 $y_{1j} - non - infected pop.$  $y_{2j} - home recoveries$  $y_{3j} - hospital recoveries$  $y_{4j} - ICU recoveries$ 

#### **Undesirable outputs**

 $b_{1j}$  – hospital deaths  $b_{2j}$  – ICU deaths



#### **Division-wise constraints** and efficiency (Example: Division 4)





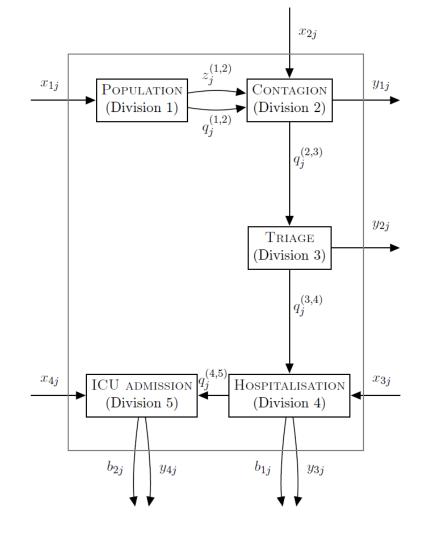
#### System-wise constraints and efficiency

#### Constraint

 $v_1^{(1)} x_{1j}^{(1)} + v_2^{(2)} x_{2j}^{(2)} + v_3^{(4)} x_{3j}^{(4)} + v_4^{(5)} x_{4j}^{(5)} - u_1^{(2)} y_{1j}^{(2)} - u_2^{(3)} y_{2j}^{(3)} - u_3^{(4)} y_{3j}^{(4)} - \left(-\hat{u}_1^{(4)} b_{1j}^{(4)}\right) - u_4^{(5)} y_{4j}^{(5)} - \left(-\hat{u}_2^{(5)} b_{2j}^{(5)}\right) \ge 0,$ 

#### Efficiency

$$E_{j} = \frac{u_{1}^{(2)}y_{1j}^{(2)} + u_{2}^{(3)}y_{2j}^{(3)} + u_{3}^{(4)}y_{3j}^{(4)} + u_{4}^{(5)}y_{4j}^{(5)}}{v_{1}^{(1)}x_{1j}^{(1)} + v_{2}^{(2)}x_{2j}^{(2)} + v_{3}^{(4)}x_{3j}^{(4)} + v_{4}^{(5)}x_{4j}^{(5)} + \hat{u}_{1}^{(4)}b_{1j}^{(4)} + \hat{u}_{2}^{(5)}b_{2j}^{(5)}}.$$





#### **Output oriented model – SOCIAL PERSPECTIVE**

 $v_1^{(1)}x_{1j_0}^{(1)} + v_2^{(2)}x_{2j_0}^{(2)} + v_3^{(4)}x_{3j_0}^{(4)} + v_4^{(5)}x_{4j_0}^{(5)} + c_0$  $\min$ subject to:  $u_1^{(2)}y_{1j_0}^{(2)} + u_2^{(3)}y_{2j_0}^{(3)} + u_3^{(4)}\alpha_{3j_0}^{(4,o)}q_{j_0}^{(3,4)} - \hat{u}_1^{(4)}\alpha_{1j_0}^{(4,o)}q_{j_0}^{(3,4)} +$  $+ u_4^{(5)} \alpha_{4j_0}^{(5,o)} q_{j_0}^{(4,5)} - \hat{u}_2^{(5)} \alpha_{2j_0}^{(5,o)} q_{j_0}^{(4,5)} = 1$  $v_1^{(1)}x_{1i}^{(1)} + v_2^{(2)}x_{2i}^{(2)} + v_3^{(4)}x_{3i}^{(4)} + v_4^{(5)}x_{4i}^{(5)} -u_1^{(2)}y_{1j}^{(2)} - u_2^{(3)}y_{2j}^{(3)} - u_3^{(4)}\alpha_{3j}^{(4,o)}q_j^{(3,4)} + \hat{u}_1^{(4)}\alpha_{1j}^{(4,o)}q_j^{(3,4)} -u_4^{(5)}\alpha_{4i}^{(5,o)}q_i^{(4,5)} + \hat{u}_2^{(5)}\alpha_{2i}^{(5,o)}q_i^{(4,5)} + c_0 \ge 0$  $j=1,\ldots,n$  $v_1^{(1)}x_{1i}^{(1)} - \hat{w}^{(1_z)}z_i^{(1,2)} + \hat{w}^{(1_q)}q_i^{(1,2)} + c_0 \ge 0,$  $i = 1, \ldots, n$  $w^{(2_z)}z_i^{(1,2)} - w^{(2_q)}q_i^{(1,2)} + v_2^{(2)}x_{2i}^{(2)} + \hat{w}^{(2)}q_i^{(2,3)} -u_1^{(2)}y_{1i}^{(2)} + c_0 \ge 0,$  $j = 1, \ldots, n$  $-w^{(3)}q_i^{(2,3)} + \hat{w}^{(3)}q_i^{(3,4)} - u_2^{(3)}y_{2i}^{(3)} + c_0 \ge 0,$  $j = 1, \ldots, n$  $-w^{(4)}q_{i}^{(3,4)}+v_{3}^{(4)}x_{3i}^{(4)}+\hat{w}^{(4)}q_{i}^{(4,5)}-u_{3}^{(4)}\alpha_{3i}^{(4,o)}q_{i}^{(3,4)}+$  $+ \hat{u}_{1}^{(4)} \alpha_{1i}^{(4,o)} q_{i}^{(3,4)} + c_0 \ge 0,$  $j = 1, \ldots, n$  $-w^{(5)}q_{i}^{(4,5)}+v_{4}^{(5)}x_{4i}^{(5)}-u_{4}^{(5)}\alpha_{4i}^{(5,o)}q_{i}^{(4,5)}+\hat{u}_{2}^{(5)}\alpha_{2i}^{(5,o)}q_{i}^{(4,5)}+$  $i = 1, \ldots, n$  $+c_0 \ge 0$ .  $c_0, \hat{w}^{(1_q)}, w^{(2_q)} \hat{w}^{(2)}, \hat{w}^{(3)}, w^{(3)}, \hat{w}^{(4)}, \hat{u}_1^{(4)}, w^{(4)}, \hat{u}_2^{(5)},$ and  $w^{(5)}$  are free  $\hat{w}^{(1_z)}, v_1^{(1)}, w^{(2_z)}, u_1^{(2)}, v_2^{(2)}, u_2^{(3)}, u_3^{(4)}, v_3^{(4)}, u_4^{(5)}, v_4^{(5)} \ge \varepsilon$ 

where  $c_0$  is the intercept and  $\varepsilon = 0.001$ .



#### Input oriented model – ECONOMIC PERSPECTIVE

 $u_1^{(2)}y_{1j_0}^{(2)} + u_2^{(3)}y_{2j_0}^{(3)} + u_3^{(4)}\alpha_{3j_0}^{(4,o)}q_{j_0}^{(3,4)} - \hat{u}_1^{(4)}\alpha_{1j_0}^{(4,o)}q_{j_0}^{(3,4)} +$  $\max$  $+ u_{4}^{(5)} \alpha_{4i_{0}}^{(5,o)} q_{i_{0}}^{(4,5)} - \hat{u}_{2}^{(5)} \alpha_{2i_{0}}^{(5,o)} q_{i_{0}}^{(4,5)} + c_{0}$ subject to:  $v_1^{(1)}x_{1i_0}^{(1)} + v_2^{(2)}x_{2i_0}^{(2)} + v_3^{(4)}x_{3i_0}^{(4)} + v_4^{(5)}x_{4i_0}^{(5)} = 1$  $-v_1^{(1)}x_{1i}^{(1)} - v_2^{(2)}x_{2i}^{(2)} - v_3^{(4)}x_{3i}^{(4)} - v_4^{(5)}x_{4i}^{(5)} +$  $+ u_1^{(2)} y_{1i}^{(2)} + u_2^{(3)} y_{2i}^{(3)} + u_3^{(4)} \alpha_{3i}^{(4,o)} q_i^{(3,4)} - \hat{u}_1^{(4)} \alpha_{1i}^{(4,o)} q_i^{(3,4)} +$  $+ u_4^{(5)} \alpha_{4i}^{(5,o)} q_i^{(4,5)} - \hat{u}_2^{(5)} \alpha_{2i}^{(5,o)} q_i^{(4,5)} + c_0 \leqslant 0$  $j = 1, \ldots, n$  $-v_1^{(1)}x_{1i}^{(1)} + \hat{w}^{(1_z)}z_i^{(1,2)} - \hat{w}^{(1_q)}q_i^{(1,2)} + c_0 \leqslant 0,$  $j = 1, \ldots, n$  $-w^{(2_z)}z_i^{(1,2)} + w^{(2_q)}q_i^{(1,2)} - v_2^{(2)}x_{2i}^{(2)} - \hat{w}^{(2)}q_i^{(2,3)} +$  $+ u_1^{(2)} y_{1i}^{(2)} + c_0 \leq 0,$  $j = 1, \ldots, n$  $w^{(3)}q_i^{(2,3)} - \hat{w}^{(3)}q_i^{(3,4)} + u_2^{(3)}y_{2i}^{(3)} + c_0 \leqslant 0,$  $j = 1, \ldots, n$  $w^{(4)}q_i^{(3,4)} - v_3^{(4)}x_{3i}^{(4)} - \hat{w}^{(4)}q_i^{(4,5)} + u_3^{(4)}\alpha_{3i}^{(4,o)}q_i^{(3,4)} -\hat{u}_{1}^{(4)}\alpha_{1i}^{(4,o)}q_{i}^{(3,4)}+c_{0}\leqslant0,$  $j=1,\ldots,n$  $w^{(5)}q_i^{(4,5)} - v_4^{(5)}x_{4i}^{(5)} + u_4^{(5)}\alpha_{4i}^{(5,o)}q_i^{(4,5)} - \hat{u}_2^{(5)}\alpha_{2i}^{(5,o)}q_i^{(4,5)} +$  $+c_0 \leq 0,$  $j=1,\ldots,n$  $c_0, \hat{w}^{(1_q)}, w^{(2_q)} \hat{w}^{(2)}, \hat{w}^{(3)}, w^{(3)}, \hat{w}^{(4)}, \hat{u}_1^{(4)}, w^{(4)}, \hat{u}_2^{(5)},$ and  $w^{(5)}$  are free  $\hat{w}^{(1_z)}, v_1^{(1)}, w^{(2_z)}, u_1^{(2)}, v_2^{(2)}, u_2^{(3)}, u_3^{(4)}, v_3^{(4)}, u_4^{(5)}, v_4^{(5)} \ge \varepsilon.$ 



### Dealing with missing data

- Some values for  $x_{1j}$ ,  $x_{2j}$ ,  $x_{3j}$ ,  $x_{4j}$ ,  $q_j^{(3,4)}$ ,  $q_j^{(4,5)}$  and  $y_{1j}$  have been estimated through linear regression;
- Values for  $y_{3j}$ ,  $y_{4j}$ ,  $b_{1j}$ , and  $b_{2j}$  have been rewritten as a function of the intermediate products that generate them in a given division p, weighted by a simulated value  $\alpha^{(p,o)}$  (where o denotes the operational environment):

$$\begin{cases} y_{3j} = \alpha_{3j}^{(4,o)} q_{j}^{(3,4)}, & \text{where:} \\ b_{1j} = \alpha_{1j}^{(4,o)} q_{j}^{(3,4)}, & \alpha_{3j}^{(4,o)} + \frac{q_{j}^{(4,5)}}{q_{j}^{(3,4)}} = 1 & \alpha_{1j}^{(4,o)} \sim U(0.12, 0.22) \text{ (Piroth et al., 2020)} \\ y_{4j} = \alpha_{4j}^{(5,o)} q_{j}^{(4,5)}, & \alpha_{3j}^{(4,o)} + \alpha_{1j}^{(4,o)} + \frac{q_{j}^{(4,5)}}{q_{j}^{(3,4)}} = 1 & \alpha_{1j}^{(4,o)} \sim U(0.12, 0.22) \text{ (Piroth et al., 2020)} \\ b_{2j} = \alpha_{2j}^{(5,o)} q_{j}^{(4,5)}, & \alpha_{4j}^{(5,o)} + \alpha_{2j}^{(5,o)} = 1 & \alpha_{2j}^{(5,o)} \sim U(0.72, 0.82) \text{ (Rahim et al., 2020)} \end{cases}$$



#### **Case study – Countries**

• A total of 55 countries, modelled as DMUs, have been assessed for 2020:

OECD Members (37)			OECD prospective members (6)	Other countries (8)
Australia	Hungary	Poland	Argentina	Algeria
Austria	Iceland	Portugal	Brazil	Egypt
Belgium	Ireland	Slovakia	Bulgaria	Morocco
Canada	Israel	Slovenia	Croatia	Russia
Chile	Italy	South Korea	Peru	Thailand
Colombia	Japan	Spain	Romania	Tunisia
Czechia	Latvia	Sweden		Ukraine
Denmark	Lithuania	Switzerland	OECD key partners (4)	Vietnam
Estonia	Luxembourg	Turkey	China	
Finland	Mexico	United Kingdom	India	
France	Netherlands	United States	Indonesia	
Germany	New Zealand		South Africa	
Greece	Norway			

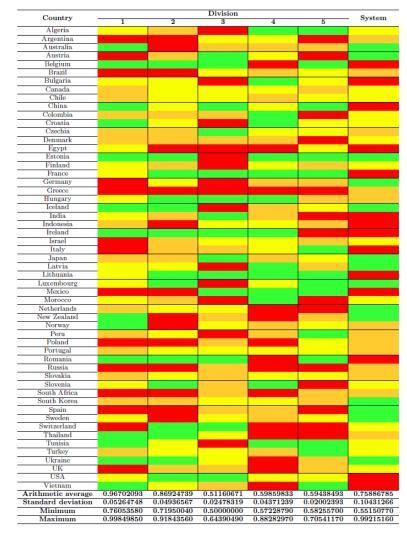


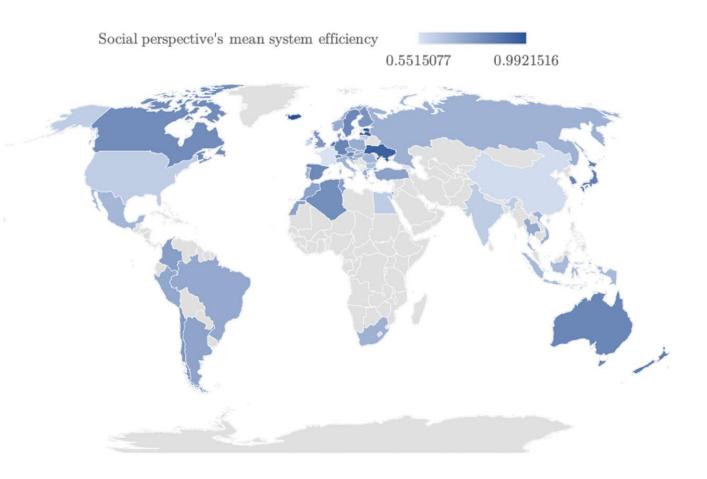
#### **Case study – Descriptive statistics**

Indicators	Average	Standard deviation	Minimum	Maximum
$x_{1j} \ (M \in)$	$123,\!447$	$415,\!134$	1,764	3,017,112
$x_{2j} \in$	$2,\!600,\!849,\!323$	$4,\!974,\!551,\!976$	$11,\!635,\!269$	$2,\!9036,\!229,\!752$
$x_{3j} \in$	$234,\!017,\!589$	$395,\!991,\!994$	13,316,280 €	$2,\!505,\!707,\!645$
$x_{4j} \in$	$477,\!753,\!640$	$773,\!360,\!577$	$9,\!225,\!565$	$4,\!975,\!291,\!796$
$z_{j}^{(1,2)}$	$64,\!463,\!704$	$165,\!860,\!132$	$192,\!889$	897,002,850
$q_{j}^{(1,2)}$	$33,\!376,\!536$	$101,\!392,\!442$	$119,\!435$	$590,\!122,\!748$
$q_{i}^{(2,3)}$	$296,\!877$	$1,\!117,\!458$	55	8,035,801
$q_{i}^{(3,4)}$	$11,\!224$	20,827	5	$125,\!379$
$q_j^{(4,5)}$	1,726	$4,\!125$	1	$29,\!179$
$y_{1j}$	$96,\!495,\!080$	$264,\!351,\!979$	$335,\!489$	$1,\!439,\!236,\!705$
$y_{2j}$	$285,\!653$	$1,\!100,\!397$	50	7,910,422



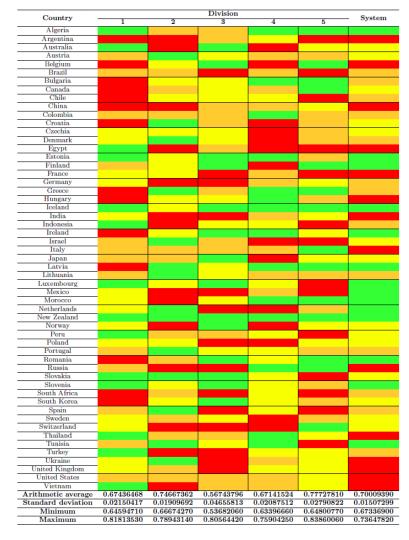
#### **Case study – Results – SOCIAL PERSPECTIVE**

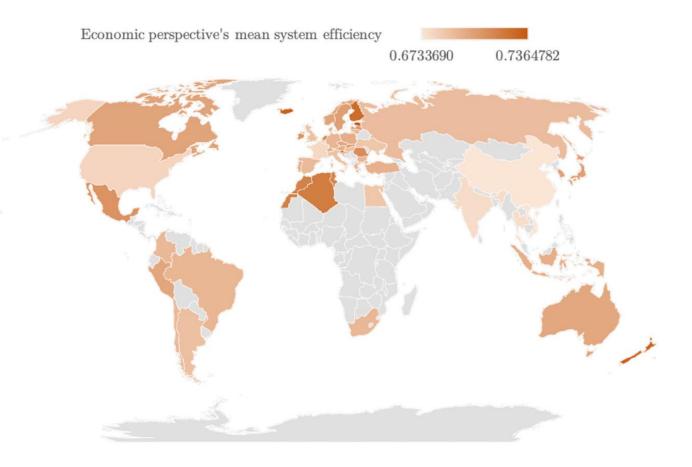






#### **Case study – Results – ECONOMIC PERSPECTIVE**







#### **Case study – Results – AGGREGATE PERSPECTIVE**

Social perspective

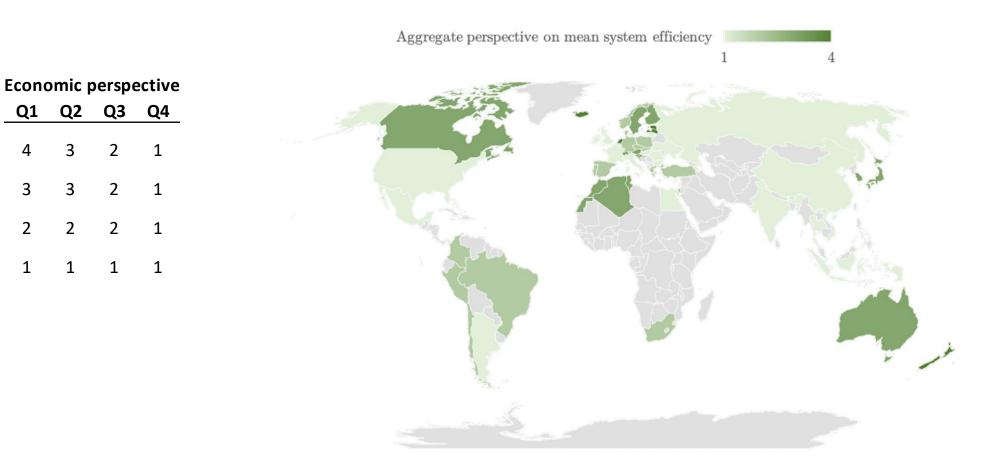
Q1

Q2

Q3

Q4

2 2 1





#### Conclusions

- None of the DMUs is systematically efficient on average in any of the considered perspectives – SOCIAL and ECONOMIC;
- Estonia, Iceland, Latvia, Luxembourg, the Netherlands, and New Zealand (≈10% of the sample) present high mean system efficiency scores in the AGGREGATE PERSPECTIVE (Q1 level);
- A total of 20 countries (≈36% of the sample) present low mean system efficiency scores in the AGGREGATE PERSPECTIVE (Q4 level);



## **Conclusions (cont.)**

- Australia, Austria, Estonia, Germany, Iceland, Japan, Latvia, Luxembourg, the Netherlands, New Zealand, South Korea, Spain, Sweden, and Ukraine (≈25% of the sample) present high mean system efficiency scores in the SOCIAL PERSPECTIVE (Q1 level);
- Another 14 countries (≈25% of the sample) present low mean system efficiency scores in the SOCIAL PERSPECTIVE (Q4 level);



## **Conclusions (cont.)**

- Algeria, Estonia, Finland, Iceland, Ireland, Latvia, Luxembourg, Mexico, Morocco, the Netherlands, New Zealand, Romania, Slovenia, and Tunisia (≈25% of the sample) present high mean system efficiency scores in the ECONOMIC PERSPECTIVE (Q1 level);
- Another 14 countries (≈25% of the sample) present low mean system efficiency scores in the ECONOMIC PERSPECTIVE (Q4 level).



#### References

Piroth, L., Cottenet, J., Mariet, A.-S., Bonniaud, P., Blot, M., Tubert-Bitter, P., & Quantin, C. (2020). Comparison of the characteristics, morbidity, and mortality of COVID-19 and seasonal influenza: A nationwide, population-based retrospective cohort study. *The Lancet Respiratory Medicine*. doi:10.1016/S2213-2600(20)30527-0

Rahim, F., Amin, S., Noor, M., Bahadur, S., Gul, H., Mahmood, A., Usman, M., Khan, M. A., Ullah, R., & Shahab, K. (2020). Mortality of patients with severe COVID-19 in the Intensive Care Unit: An observational study from a major COVID-19 receiving hospital. *Cureus*. doi:10.7759/cureus.10906

Rees, E. M., Nightingale, E. S., Jafari, Y., Waterlow, N. R., Cliord, S., Carl, C. A., Group, C. W., Jombart, T., Procter, S. R., & Knight, G. M. (2020). COVID-19 length of hospital stay: A systematic review and data synthesis. *BMC Medicine*, *18*. doi:10.1186/s12916-020-01726-3

Remuzzi, A., & Remuzzi, G. (2020). COVID-19 and Italy: what next? *The Lancet, 395*, 1225-1228. doi:10.1016/S0140-6736(20)30627-9



## Thank you!

FCT Project PTDC/EGE-OGE/30546/2017 duarte.dinis@tecnico.ulisboa.pt